Nature of the $f_0(600)$ from its N_c dependence at two loops in unitarized Chiral Perturbation Theory

J. R. Peláez and G. Ríos
Departamento de Física Teórica II, Universidad Complutense de Madrid, 28040 Madrid, Spain

By using unitarized two-loop Chiral Perturbation Theory partial waves to describe pion-pion scattering we find that the dominant component of the lightest scalar meson does not follow the $\bar{q}q$ dependence on the number of colors that, in contrast, is obeyed by the lightest vectors. The method suggests that a subdominant $\bar{q}q$ component of the $f_0(600)$ possibly originates around 1 GeV.

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The lightest scalar mesons are a subject of a longstanding controversy that is recently receiving relevant contributions that could help settling the questions about their existence and nature. Experimentally, several analyses [1], find poles for the $f_0(600)$ and κ, the lightest scalars with isospin 0 and 1/2, respectively. The former is of interest for spectroscopy but also for understanding spontaneous chiral symmetry breaking, since it has precisely the vacuum quantum numbers. On the theoretical side, the QCD chiral symmetry breaking pattern has been shown to lead to $f_0(600)$ and κ poles in $\pi\pi$ and πK scattering [2, 3, 4, 5, 6]. Concerning the spectroscopic classification, the caveat for most hadronic models is the difficulty to extract the quark and gluon composition without assumptions hard to justify within QCD. In contrast, when using fundamental degrees of freedom, i.e., in lattice or with QCD inspired potentials, other complications arise, related to chiral symmetry breaking, the use of actual quark masses or the physical pion or kaon masses and decay constants. All approaches are also complicated by the possible mixing of different states in the physical one. Most of these caveats are overcome in a recent approach [7] ([8] for a review) based on the pole dependence on the number of colors, N_c, of meson-meson scattering within unitarized Chiral Perturbation Theory.

The relevance of the large N_c expansion [9] is that it provides an analytic approximation to QCD in the whole energy region and a clear identification of $\bar{q}q$ states, that become bound states as $N_c \to \infty$, and whose masses scale as $O(1)$ and their widths as $O(1/N_c)$. Other kind of hadronic states may show different behaviors [10].

In order to avoid any spurious N_c dependence in the hadronic description, we use Chiral Perturbation Theory (ChPT), which is the QCD low energy Effective Theory, and where the large N_c behavior is implemented systematically. It is built as the most general derivative expansion of a Lagrangian [11], in terms of π, K and η mesons compatible with the QCD symmetries, These particles are the Goldstone bosons associated to the spontaneous chiral symmetry breaking of massless QCD and are therefore the lightest degrees of freedom. Actually, the u, d and s quark masses are non-vanishing but small enough to be treated as perturbations that give rise to π, K and η masses. Thus, ChPT is an expansion in powers of momenta and masses and, generically, its applicability is limited to a few hundred MeV above threshold. Each order is made of all possible terms multiplied by a "chiral" parameter. These Low Energy Constants (LECS) are renormalized to absorb loop divergences order by order, and once determined from experiment they can be used in any other pseudo-Goldstone boson amplitude.

For our purposes, we are interested in meson-meson scattering amplitudes, since by unitarization they generate dynamically resonances not initially present in ChPT [4, 5, 6, 12, 13, 14]. Indeed [8, 14], with the coupled channel Inverse Amplitude Method (IAM), the one loop ChPT meson-meson amplitudes describe data up to roughly $\sqrt{s} \simeq 1.2\,\text{GeV}$ and generate the ρ and K^* vectors, as well as the $f_0(600)$, κ, $a_0(980)$ and $f_0(980)$ scalars, and, most importantly, using LECS compatible with standard ChPT and therefore *without any further assumption or source of spurious N_c dependence.*

By scaling the one-loop ChPT parameters with their N_c behavior, it was recently shown that the generated ρ and K^* show the typical N_c behavior of $\bar{q}q$ states, whereas the scalars are at odds with a dominant $\bar{q}q$ component. These results, confirmed by other methods [15], implied some cancellation between tree level diagrams proportional to LECS, and that loop diagrams with two intermediate mesons are very relevant in the generation of light scalars. But such loop diagrams are subdominant in the large N_c counting and one could wonder about the stability under small changes in the LECS and about higher order ChPT corrections that could become larger than loop terms at sufficiently large N_c, and reveal the existence of subdominant $\bar{q}q$ components.

Here we present a method to quantify the above statements, and generalize the approach of [7] to two-loop ChPT and in particular to $\pi\pi$ scattering [16]. Despite the many second order parameters and their large uncertainties, the data can be well described and we find once more that the $\rho(770)$ behaves as $\bar{q}q$ with N_c, whereas the $f_0(600)$ main component does not behave as such. Furthermore, with the second order calculation a dominant $\bar{q}q$ behavior cannot be imposed on the $f_0(600)$ and the ρ simultaneously, but a subdominant $\bar{q}q$ $f_0(600)$ compo-

	$O(p^4)$ LECS				$O(p^6)$ LECS			
Refs.	[11]	[18]	[8]	we use	[17]	[16]	[19]	we use
$10^3 l_1^r$	-6.0	-5.4	-3.5	3.5±2.2	-3.3	-5.2	-4.6	-3.3±2.2
$10^3 l_2^r$	5.5	5.7	4.7	4.7±1.0	2.9	2.3	2.0	2.9±1.0
$10^3 l_3^r$	0.82	0.82	-2.6	0.82±3.8	1.2	0.82	0.82	0.82±3.8
$10^3 l_4^r$	5.6	5.6	8.6	6.2±2.0	2.4	5.6	6.2	6.2±2.0

TABLE I: Sample of LECS central values. The fifth and ninth columns, whose uncertainties roughly represent the previous sample, are used in our fits in the text to calculate χ^2_{LECS}.

nent seems to arise at larger N_c around 1 GeV.

Thus, at leading order, the only parameter is the pion decay constant in the chiral limit, $f_0 = O(\sqrt{N_c})$, fixed by the spontaneous symmetry breaking scale $4\pi f_0 \simeq 1\,$GeV. Indeed, ChPT $\pi\pi$ scattering amplitudes are expanded as $t \simeq t_2 + t_4 + t_6 + \ldots$ with $t_k = O((p/4\pi f_0)^k)$ and are, generically, $O(1/N_c)$. In particular, the LECS appearing in $\pi\pi$ scattering at $O(p^4)$ [11], all scale as $O(N_c)$. For simplicity we use the $SU(2)$ notation, l_i^r, since we are only dealing with $\pi\pi$ scattering (see the last reference in [11] for a translation to $SU(3)$). In Table I we give a sample of l_i^r sets given in the literature, whose differences we take as systematic uncertainties for the set we use in our fits below. In Table II we also list the six $O(p^6)$ constants that appear in $\pi\pi$ scattering, denoted r_i. They all count as $O(N_c^2)$. Those values are just *estimates* assuming they are saturated by the multiplets of the lightest (predominantly vector) resonances. This hypothesis works well at $O(p^4)$ [20], but for $O(p^6)$ is probably just correct within an order of magnitude [16] and we conservatively assign a 100% uncertainty.

The large N_c counting does not specify at what renormalization scale μ it applies, thus becoming an uncertainty studied in [7] for the one-loop LECS. For the r_i, the scale dependence is much more cumbersome and has not been written explicitly. Nevertheless, in both cases it is subleading in $1/N_c$, and given the fact that we have a 100% error on the r_i should be well within errors for our fits. Hence, we do not perform such analysis here, simply setting $\mu = 770\,$MeV, as usual.

Next, resonances can be found as poles in partial wave amplitudes t_{IJ} of isospin I and angular momentum J that, in the elastic regime satisfy the unitarity condition:

$$\mathrm{Im}\, t = \sigma |t|^2 \Rightarrow \mathrm{Im}\,\frac{1}{t} = -\sigma \Rightarrow t = \frac{1}{\mathrm{Re}\, t^{-1} - i\sigma}, \qquad (1)$$

where σ is the known two-meson phase space and we have omitted the IJ indices for brevity. Note that ChPT expansions violate *exact* unitarity, since in the first Eq.(1), the highest power of momenta on the right hand is twice that on the left. Unitarity is only satisfied *perturbatively*

$$\mathrm{Im}\, t_2 = 0, \quad \mathrm{Im}\, t_4 = \sigma t_2^2, \quad \mathrm{Im}\, t_6 = \sigma 2 t_2 \mathrm{Re}\, t_4, \ \ldots \qquad (2)$$

If we replace in Eq.(1) $\mathrm{Re}\, t^{-1}$ by its ChPT approximation we get the Inverse Amplitude Method (IAM), that satisfies elastic unitarity exactly. At $O(p^4)$ it reads,

$$t \simeq t_2^2/(t_2 - t_4), \qquad (3)$$

and its fit to "data only" is listed in Table II, in the SU(2) notation. The fit quality is remarkable, given the huge systematic uncertainties, (conservatively $\pm 5^0$ and 5% error for the $f_0(600)$ channel) and we refer to [8, 14] for details and figures with a comparison with data. Using Eqs.(1) and (2), the $O(p^6)$ IAM [4, 21] reads

$$t \simeq t_2^2/(t_2 - t_4 + t_4^2/t_2 - t_6), \qquad (4)$$

that recovers the $O(p^6)$ ChPT expansion at low energies and describes well elastic $\pi\pi$ scattering data [21]. In addition, the IAM has a right cut that defines two Riemann sheets. In the second sheet we find poles associated to resonances; in particular, for the $\rho(770)$ in the $(I, J) = (1, 1)$ channel and for the $f_0(600)$ in the $(0, 0)$ one. For narrow resonances, $\Gamma << M$, the pole position is related to its mass and width as $\sqrt{s_{\rm pole}} \simeq M - i\Gamma/2$, and we keep this as a *definition* for the wide $f_0(600)$, whose $M \sim 400 - 500\,$MeV and $\Gamma \sim 400 - 600\,$MeV.

By scaling the previous parameters with their dominant N_c behavior, namely, $f_{N_c} \to f\sqrt{N_c/3}$, $l^r_{i,N_c} \to l_i^r N_c/3$ and $r_{i,N_c} \to r_i(N_c/3)^2$, we obtain the large N_c dependence of M_{N_c} and Γ_{N_c} of the ρ and $f_0(600)$ poles generated by the IAM. If a resonance is predominantly a $\bar{q}q$, $M_N \sim O(1)$ and $\Gamma_N \sim O(1/N_c)$, and so it was shown [7] that the $O(p^4)$ IAM reproduced remarkably well that behavior for the $\rho(770)$ and $K^*(892)$, two well established $\bar{q}q$ mesons. This is the expected behavior if in Eq.(3) one neglects the two-meson loop terms, which are subleading at large N_c with respect to $O(p^4)$ LECS contributions.

In contrast, the lightest scalars follow a qualitatively different behavior. Loop diagrams, instead of the $O(p^4)$ LECS terms, play a relevant role in determining the scalar pole position. This is nothing but the well known fact that light scalars are dynamically generated by the resummation in Eq.(3) of two-meson loop diagrams [4, 5, 6, 13, 14]. However, although relevant at $N_c = 3$, loop diagrams are suppressed by $1/N_c$ compared to tree level terms with LECS, and the $O(p^6)$ terms could become bigger at some larger N_c, where the $O(p^4)$ N_c results should no longer be trusted. For that reason it is important to check the $O(p^6)$ IAM: it should give small corrections to the $O(p^4)$ close to $N_c = 3$, but it may deviate at larger N_c and even unveil some subdominant $\bar{q}q$ component.

Still, before scaling the $O(p^6)$ IAM, let us first note that $M_{N_c} = O(1)$ and $\Gamma_{N_c} = O(1/N_c)$ is only the *leading* $\bar{q}q$ scaling. Taking into account subleading uncertainties, to consider a resonance a $\bar{q}q$ state, it is enough that

$$M^{\bar{q}q}_{N_c} = \widetilde{M}\left(1 + \frac{\epsilon_M}{N_c}\right), \ \Gamma^{\bar{q}q}_{N_c} = \frac{\widetilde{\Gamma}}{N_c}\left(1 + \frac{\epsilon_\Gamma}{N_c}\right), \qquad (5)$$

were $\widetilde{M}$ and $\widetilde{\Gamma}$ are unknown but N_c independent and the subleading terms have been gathered in $\epsilon_M, \epsilon_\Gamma$, which are $O(1)$. Thus, for a $\bar{q}q$ state, the *expected* M_{N_c} and Γ_{N_c} can be obtained from those at $N_c - 1$ generated by the IAM,

$$M^{\bar{q}q}_{N_c} \simeq M_{N_c-1}\left[1+\epsilon_M\left(\frac{1}{N_c}-\frac{1}{N_c-1}\right)\right] \equiv M_{N_c-1}+\Delta M^{\bar{q}q}_{N_c}, \tag{6}$$

$$\Gamma^{\bar{q}q}_{N_c} \simeq \frac{\Gamma_{N_c-1}\,(N_c-1)}{N_c}\left[1+\epsilon_\Gamma\left(\frac{1}{N_c}-\frac{1}{N_c-1}\right)\right] \equiv \frac{\Gamma_{N_c-1}\,(N_c-1)}{N_c}+\Delta\Gamma^{\bar{q}q}_{N_c}. \tag{7}$$

Note the $\bar{q}q$ index for all quantities obtained *assuming* a $\bar{q}q$ behavior. We refer the values at N_c to those at N_c-1 because then we will be able to calculate from what N_c value a resonance starts behaving as a $\bar{q}q$, which is of interest in order to look for subdominant $\bar{q}q$ components. Thus, we can now define an *averaged* $\bar{\chi}^2_{\bar{q}q}$to measure how close a resonance is to a $\bar{q}q$ behavior, using as uncertainty the $\Delta M^{\bar{q}q}_{N_c}$ and $\Delta\Gamma^{\bar{q}q}_{N_c}$.

$$\bar{\chi}^2_{\bar{q}q}=\frac{1}{2n}\sum_{N_c=4}^{n}\left[\left(\frac{M^{\bar{q}q}_{N_c}-M_{N_c}}{\Delta M^{\bar{q}q}_{N_c}}\right)^2+\left(\frac{\Gamma^{\bar{q}q}_{N_c}-\Gamma_{N_c}}{\Delta\Gamma^{\bar{q}q}_{N_c}}\right)^2\right] \tag{8}$$

Since at $N_c = 3$ we expect *generically* 30% uncertainties we take $\epsilon_M = \epsilon_\Gamma = 1$. Let us note that ΔM, and even faster $\Delta\Gamma$, tend to zero for large N_c and eventually become smaller than our precision determining the pole position, 1 MeV, which we add as a systematic error. When a state is predominantly $\bar{q}q$, it should follow Eq.(5) and $\bar{\chi}^2_{\bar{q}q} \lesssim 1$. Otherwise $\bar{\chi}^2_{\bar{q}q} \gg 1$. Note that n should not be too far from 3, since we are looking for the N_c behavior of the physical state. If we took n too large, we could be changing radically the original mixture of the observed state and for sufficiently large N_c even the tiniest $\bar{q}q$ component could become dominant over the rest [22]. Therefore, our method first determines the behavior of the resonance *dominant component*, but also, when $\bar{q}q$ is not dominant, the N_c at which it becomes so.

Furthermore, by minimizing its $\bar{\chi}^2_{\bar{q}q}$ we can constrain a state to follow the $\bar{q}q$ behavior. Thus, we will minimize $\chi^2_{data} + \chi^2_{LECS}$ plus the $\bar{\chi}^2_{\bar{q}q}$ of either of the ρ, or the $f_0(600)$, or both. The averaged χ^2_{LECS} measures how far the fitted LECS are from their typical values in the tables and stabilizes them. Note that $\pi\pi$ scattering data are poor, with large systematic uncertainties and not very sensitive to some of the individual parameters but to their combinations, thus producing large correlations, driving the LECS away from the typical values for tiny improvements in the χ^2, particularly at $O(p^6)$. We will provide the χ^2_{data}, χ^2_{LECS}, and $\bar{\chi}^2_{\bar{q}q}$, divided by the number of data points, the number of LECS and $2n$, respectively.

Thus, in Table II we show three $O(p^4)$ fits: to data only, constrained to a ρ $\bar{q}q$ hypothesis, or constraining the $f_0(600)$ to be a $\bar{q}q$. We list for each fit the different χ^2 described above and we see that our approach clearly identifies the ρ as a $\bar{q}q$, since $\bar{\chi}^2_{\rho,\bar{q}q} \simeq 0.25$. In contrast, $\bar{\chi}^2_{\bar{q}q} \geq 125$ for the $f_0(600)$, even if we constrain the fit to minimize also $\bar{\chi}^2_{\bar{q}q}$ for the $f_0(600)$ (at the price of a higher $\chi^2_{LECS} = 5.6$). This is the quantitative statement of the $O(p^4)$ results in [7, 8] where it was concluded that the main component of the $f_0(600)$ was not $\bar{q}q$.

Fit	fits $O(p^4)$ Only data	fits $O(p^4)$ ρ as $\bar{q}q$	fits $O(p^4)$ $f_0(600)$ as $\bar{q}q$	fits $O(p^6)$ ρ as $\bar{q}q$	fits $O(p^6)$ $f_0(600)$ as $\bar{q}q$	fits $O(p^6)$ $\rho, f_0(600)$ as $\bar{q}q$
$10^3 l_1$	-3.8	-3.8	-3.9	-5.4	-5.7	-5.7
$10^3 l_2$	4.9	5.0	4.6	1.8	2.6	2.5
$10^3 l_3$	0.43	0.42	2.6	1.5	-1.7	0.39
$10^3 l_4$	7.2	6.4	15	9.0	1.7	3.5
χ^2_{data}	**1.1**	**1.2**	**1.4**	**1.1**	**1.4**	**1.5**
χ^2_{LECS}	**0.08**	**0.03**	**5.6**	**1.9**	**2.1**	**1.4**
$\bar{\chi}^2_{\rho,\bar{q}q}$	0.26	**0.22**	0.32	**0.93**	2.0	**1.3**
$\bar{\chi}^2_{f_0(600),\bar{q}q}$	140	143	**125**	15	**3.5**	**4.0**
$10^4 r_1$		-0.6		-0.60	-0.60	-0.58
$10^4 r_2$		1.3	Our	1.5	1.3	1.5
$10^4 r_3$	Ref.	-1.7	$O(p^6)$	-1.4	-4.4	-3.2
$10^4 r_4$	[16]	-1.0	fits	1.4	-0.03	-0.49
$10^4 r_5$		1.1		2.4	2.7	2.7
$10^4 r_6$		0.3		-0.60	-0.70	-0.62

TABLE II: IAM fits to data or constrained to a $\bar{q}q$ N_c behavior for the ρ and $f_0(600)$. In boldface the χ^2 minimized on each fit. For the $O(p^6)$ fits we also provide the r_i estimates [16]

Unfortunately, the $O(p^6)$ analysis has a large freedom and thus χ^2_{LECS} plays a relevant role to stabilize the fit, but keeping in mind that the r_i uncertainties were arbitrarily chosen to be 100%. In Table II and Fig. 1 we show three $O(p^6)$ fits: constraining the ρ as a $\bar{q}q$ (Fig. 1. Top), or the $f_0(600)$ (Fig. 1. Center) or both (Fig. 1, Bottom). As expected, the $O(p^6)$ results are consistent with those at $O(p^4)$ not far from $N_c = 3$ [7, 8] but for the scalar channel they deviate around $N_c \sim 8$.

In particular, in the "ρ as $\bar{q}q$" fit a $\bar{q}q$ dominant nature comes out neatly for the ρ, whose $\bar{\chi}^2_{\bar{q}q} \sim 0.9$, but is discarded for the $f_0(600)$, since its $\bar{\chi}^2_{\bar{q}q} \simeq 15$ and Fig.1 shows that its mass and width *both rise* when N_c increases *not too far from real life,* $N_c = 3$. However, for $N_c > 8$ *the mass tends to a constant around 1 GeV and the width decreases*, but not with a $1/N_c$ scaling. *This suggests a mixing with a $\bar{q}q$ subdominant component*, arising as loop-diagrams become more suppressed at large N_c.

One might wonder if the $f_0(600)$ could also be forced to behave predominantly as a $\bar{q}q$. Thus we made a "$f_0(600)$ as a $\bar{q}q$" constrained fit (Fig. 1, Center). The price to pay is a deterioration of the χ^2_{data} and an unacceptable ρ $\bar{q}q$ behavior, since its $\bar{\chi}^2_{\bar{q}q} \sim 2$. Still, the $f_0(600)$ $\bar{\chi}^2_{\bar{q}q}$ decreases only to 3.5 (for 34 N_c points). This extreme case allows us to conclude that *the $O(p^6)$ calculation cannot accommodate a $\bar{q}q$ dominant component for the $f_0(600)$*.

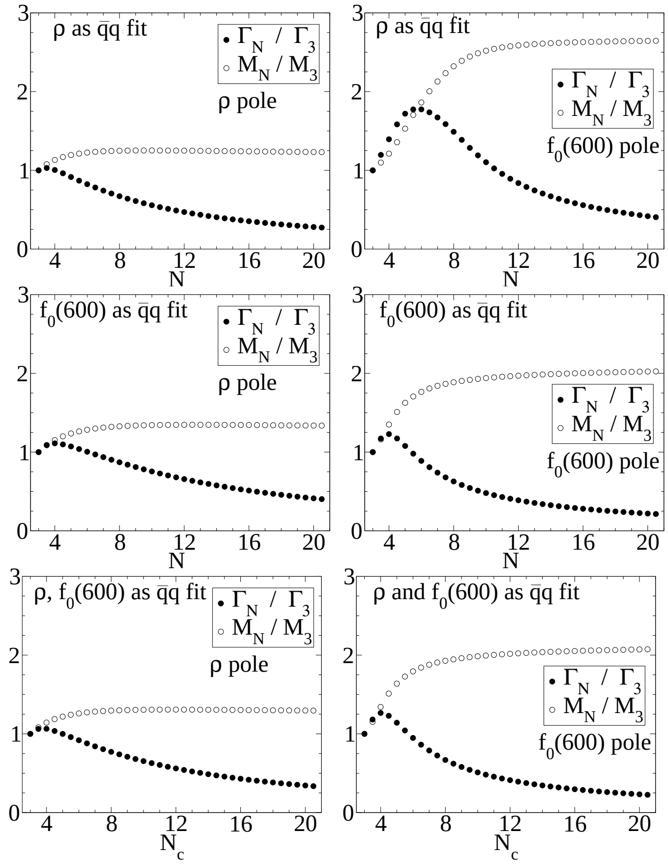

FIG. 1: Mass and width N_c behavior of the ρ and $f_0(600)$ from an $O(p^6)$ IAM data fit minimizing *also* the $\bar{\chi}^2_{\bar{q}q}$: of the ρ (Top). of the $f_0(600)$ instead of the ρ (Center) of both the $f_0(600)$ and ρ (Bottom).

Finally, we have studied how much of a *subdominant* $\bar{q}q$ behavior the $f_0(600)$ can accommodate without spoiling that of the ρ. Hence, we have also minimized in the fit the $\bar{\chi}^2_{\bar{q}q}$ both for the ρ and $f_0(600)$ (Fig. 1, Bottom). The $f_0(600)$ still does not behave predominantly as a $\bar{q}q$, since its $\bar{\chi}^2_{\bar{q}q} \simeq 4$. However, it starts behaving as a $\bar{q}q$, i.e., $\bar{\chi}^2_{\bar{q}q} \simeq 1$, for $N_c \geq 6$. The $\bar{q}q$ behavior of the ρ only deteriorates a little, $\bar{\chi}^2_{\bar{q}q} \simeq 1.3$, and should not be pushed much further. This result suggests that the subdominant mixing with a $\bar{q}q$ state around 1 GeV seen in the first fit, would become dominant around $N_c > 6$, *at best* .

In summary, we have presented a method to determine quantitatively how close the N_c dependence of a resonance pole is to a $\bar{q}q$ behavior. We have applied this measure to the poles generated in $\pi\pi$ scattering by unitarized Chiral Perturbation Theory, which is the effective low energy theory of QCD and reproduces systematically its large N_c expansion. The method is able to confirm the $O(p^4)$ qualitative results [7, 8], identifying the ρ as a $\bar{q}q$ state and showing that the $f_0(600)$ is at odds with a dominant $\bar{q}q$ component. We have extended the method to $O(p^6)$ confirming the stability of our $O(p^4)$ conclusions, but also showing that a possible subdominant $\bar{q}q$ may originate around 1 GeV. This provides further support, based on the QCD N_c dependence, to some models that generate the $f_0(600)$ from final state meson interactions, and locate a "preexisting" $\bar{q}q$ scalar nonet [3, 5] around 1 GeV. The methods presented here should be easily generalized to investigate the nature of other dynamically generated mesons [23] and baryons [24].

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